Optimization in Academia: Minimizing Cost of Faculty Budget on Term-by-Term Basis

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Mathematics

Introduction

The focus of this report is to minimize the adjunct faculty budget for a graduate program at a small, private university on a term-by-term basis while providing high quality instruction to the students. The adjunct faculty budget in this paper consists of two components: adjunct faculty stipends and overload payments to full-time faculty if he/she is teaching a course past the load stated in the full-time faculty's contract with the university. For example, the university in the report is a teaching university where most full-time faculty teach 4 courses per term. Teaching these 4 courses per term does not affect the adjunct budget because it is part of the faculty's yearly salary. However, full-time faculty may have the opportunity to teach an additional course and receive overload compensation. This overload compensation would come out of the adjunct faculty budget. This paper develops a mathematical model for the scheduling process and uses integer programming to find the optimal solution.

The course scheduling process for the university is an iterative process that begins by using the course offerings in the term from the last academic year as a template for the course offerings in the term for the next academic year. For example, the 2017 spring semester schedule is not built from scratch but uses the 2016 spring semester schedule as a template. Although convenient, this method often leads to minimal changes and can result in program directors following patterns over time that may not be optimal in a complete sense. Rather than only making changes to the schedule if a new adjunct faculty member is hired when another faculty member leaves the university or if a new course is offered, this report provides a strategic approach by developing a model to assign faculty on a term-by-term basis to deliver optimal instruction while minimizing cost to the adjunct faculty budget. Once the model is put in place, there is opportunity to perform sensitivity analysis on adjustments of the parameters to analyze changes in the optimal solution. Lastly, the model can be used as a benchmark for how effective the current scheduling process is or as a guide to a program director new to the course scheduling process.

This report includes a total of 20 instructors; 14 adjuncts and 6 full-time faculty. The courses in the report are for the 2017 spring semester. The program director has already determined the courses on the schedule. There are 13 different courses and a total of 16 sections (3 of the courses have 2 sections). The courses cover a wide range of topics, ranging from areas in business to technical design. The optimal value of the objective function will be the minimal value in USD needed to teach courses subject to the following constraints:

- An adjunct faculty cannot teach more than 9.75 work load units (WLU). Each course carries a number of work-load units. The combination of courses an adjunct faculty is assigned to must be less than 9.75 WLU. This is a university standard set to ensure all adjunct faculty remain part-time employees.

- Full-time faculty are permitted to teach only one overload. A full-time faculty member must also be willing to teach the course as an overload before being assigned to teach the course as an overload.
- Any assignment of a full-time faculty member to a course, that is not an overload assignment, must fit within the load of a full-time faculty member as determined per the full-time faculty member's contract with the university.
- There must be one and only one faculty assigned to each course. There are no opportunities for team teaching.
- Any faculty member assigned to a course must be eligible to teach the course. For example, a faculty member may be qualified to teach a marketing course but not a supply chain course or vice versa.
- Any faculty member assigned to a course must be eligible to teach the course and be able to do so effectively. Faculty reviews completed by students will be used to make sure the quality of teaching on average in the program meets a certain standard.

Method

The report utilizes integer linear programming to achieve an optimal solution for the problem. Integer linear programming is derived from linear programming, which uses a formulated mathematical model to reach an optimal goal. The requirement of binary integer values is the only way this problem deviates from a linear programming formulation (Hillier and Lieberman 474). In this case, the optimal solution is to minimize the adjunct faculty budget while providing high quality instruction to students. Lingo software will be used to find the solution. Although it would be time consuming, the binary integer programming "branch and bound" algorithm is a way to solve the problem by hand. Branch-and-bound is an effective enumeration procedure especially known for application to integer linear program problems. A problem with a very large, finite number of feasible solutions is divided into smaller and smaller subproblems that can be more easily solved (Hillier and Lieberman 502).

Assumptions

The model satisfies the mathematical assumptions of certainty, additivity and proportionality but not divisibility since the decision variables can only take on values of 0 or 1.

Table 1. Mathematical Assumptions

Assumption	Definition	Assumption Satisfied
Certainty	Values of parameters are known constants	Satisfied
Additivity	Every function in model is the sum of individual contributions of respective activities	Satisfied
Proportionality	Contribution of each activity to the objective function is proportional to the level of activity (no exponents greater than the power of one in the model)	Satisfied
Divisibility	Decision variables can take on any value, not restricted to integers	Not Satisfied; binary integers utilized in model

(Adapted from Hillier and Lieberman 38)

Below are the assumptions in the context of the report:

- The program director has already prepared the course offerings for the 2017 spring term. It is assumed the program director prepared the optimal selection of courses for the schedule
- Faculty are determined eligible to teach a course based on his/her professional background. For example, a professor may be qualified to teach a prototyping pattern-making course but not a supply chain course.
- Stipend rates are fixed for each instructor.
- The course selection for the semester set by the program director does not change and the courses will have enough enrollment to run in the spring term.
- The maximum number of course a full-time faculty member can teach in the program at no cost because of his/her contract with the university is collected by reviewing the complete 2017 spring schedule already available at the university. The project assumes the schedule is already prepared and all other faculty assignments are known.
- The model assumes that adjunct faculty are not teaching in any other program on campus so the only work load measures for the adjunct faculty are the assignments in the program in this report.
- All faculty are assumed able to teach at the day/time whenever the courses are offered on the schedule
- Full-time faculty assigned to a course are not included in the objective function since the faculty teach these courses as part of their university contract and these funds are part of the yearly salary for the faculty, which is not part of the term-by-term adjunct faculty budget. The objective function seeks to minimize this term-by-term adjunct faculty budget.
- The term-by-term adjunct faculty budget includes adjunct stipends and overload stipends only.
- The quality rating of an instructor is fixed and does not change with the amount of courses an instructor is assigned
- Only one faculty can teach a course; there are no co-teaching or team-teaching assignments.

Model Formulation

Table 2 contains description of the variables used in the report.

Table 2. Description of variables

Variable	**Description**	**Type**
i	Course	Index variable
j	Section of course i	Index variable
k	Instructor	Index variable
C	Cost in USD to teach courses for the program in the 2017 spring term	Objective value

f_kij	Full-time professor, teaching course as part of full-time position of employment, f=1 if k is teaching section j of i, 0 otherwise	Decision variable
a_kij	Adjunct instructor, a=1 if k is teaching section j of i, 0 otherwise	Decision variable
o_kij	Overload course assignment for full-time professor, o=1 if k is teaching section j of i as overload, 0 otherwise	Decision Variable
s_k	Stipend for instructor in USD per credit. If instructor is an adjunct, the instructor is paid a stipend. If full time faculty, the instructor is due stipend only if assignment is an overload to the full-time faculty's load. Full-time faculty assignment within load are cost of $0 since they are salaried employees.	Parameter
w_k	Full-time faculty willingness to teach a course as overload assignment. w=1 if faculty willing, 0 otherwise.	Parameter
e_ki	Eligibility for faculty k to teach course i, where e=1 if faculty k is capable of teaching course i, 0 otherwise	Parameter
q_k	Quality rating of instructor, measured on scale 1-5; 1 is least effective, 5 is most effective	Parameter
m_k	Max course(s) a full-time professor can teach in the program at $0 cost per the faculty's university contract	Parameter
h_i	Workload units (WLU) for course i. As a consistent measure to track the work involved in a course, each course has a total WLU. For example, a 3.0 credit lecture course is a 3.0 WLU. WLUs vary based on course teaching method (lecture, studio, lab, by-appointment, etc.)	Parameter
c_i	Credits per course	Parameter
n_k	Number of instructors	Parameter
n_a	Number of adjuncts	Parameter
n_c	Number of courses	Parameter
n_i	Number of sections of course i	Parameter

It is important to note that instructor (k) indexes the faculty from 1-20. When representing the faculty in the model, these values are written out as text to make sure each variable is unique. For example, a_1111 could mean adjunct faculty 11, teaching course 1, section 1 or adjunct faculty 1, teaching course 11, section 1. This issue is avoided by representing the faculty as a_eleven11 (faculty eleven, teaching course 1, section 1).

Table 3 is the model formulation.

Table 3. Mathematical model consisting of objective function and nine functional constraints.

Objective function:
Min $C = \sum_{k=1}^{n\_a} \sum_{i=1}^{n\_c} \sum_{j=1}^{n\_i} s_k * a_{kij} * c_i + \sum_{n_a+1}^{k} \sum_{i=1}^{n\_c} \sum_{j=1}^{n\_i} s_k * o_{kij} * c_i$
Subject to:
1. $\sum_{i=1}^{n\_c} \sum_{j=1}^{n\_i} h_i * a_{kij} \leq 9.75\ WLU\ for\ \forall\ n_a;$
2. $\sum_{i=1}^{n_i} \sum_{j=1}^{n_c} o_{kij} \leq 1\ for\ k = n_{a+1} - 20;$
3. $\sum_{i=1}^{n\_i} \sum_{j=1}^{n\_c} f_{kij} \leq m_k\ for\ k = n_{a+1} - 20;$
4. $\sum_{k=1}^{n\_a} a_{kij} + \sum_{n_a+1}^{k} f_{kij} + \sum_{n_a+1}^{k} o_{kij} = 1\ \ \forall\ ij;$
5. $\sum_{i=1}^{n\_i} \sum_{j=1}^{n\_c} o_{kij} \leq w_k\ for\ k = n_{a+1} - 20;$
6. $\sum_{i=1}^{n_i} \sum_{j=1}^{n_c} a_{kij} \leq 5 * e_k\ for\ \forall\ n_a;$
7. $\sum_{i=1}^{n_i} \sum_{j=1}^{n_c} f_{kij} + \sum_{i=1}^{n_i} \sum_{j=1}^{n_c} o_{kij} \leq 5 * e_{ki}\ for\ k = n_{a+1} - 20;$
8. $(1/16)(\sum_{i=1}^{n_i} \sum_{j}^{n\_c} q_k * a_{kij} + q_k * f_{kij} + q_k * o_{kij}) \geq 4.25\ for\ \forall\ k$
9. All a_{kij}, f_{kij}, o_{kij} binary

The Data

Figures 1 – 5 provide the data for the report.

Figure 1. Faculty Teach Course Eligibility Matrix. x indicates faculty *i* eligible to teach course *k*

	k=1	k=2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
i=1														x						x
i=2												x								
3						x		x				x							x	
4		x										x								
5								x				x			x	x	x		x	
6				x	x		x					x			x	x	x	x		
7				x	x		x					x			x	x	x	x		
8				x	x		x					x			x	x	x	x		
9							x													
10															x					

11															x					
12					x											x	x	x		
13	x		x						x	x	x		x							

Figure 2. Data on Faculty Eligibilty to Teach Courses. Data is binary where *e_ki* = 1 if faculty *i* is eligible to teach course *k*; 0 otherwise. Only faculty 1 and 2 displayed due to size of data.

e_ki				
e_one1	0		e_two1	0
e_one2	0		e_two2	0
e_one3	0		e_two3	0
e_one4	0		e_two4	1
e_one5	0		e_two5	0
e_one6	0		e_two6	0
e_one7	0		e_two7	0
e_one8	0		e_two8	0
e_one9	0		e_two9	0
e_one10	0		e_two10	0
e_one11	0		e_two11	0
e_one12	0		e_two12	0
e_one13	1		e_two13	0

Figure 3. Data on sections of courses with credits and work-load units

course	**i**	**j**	**c_i**	**h_i**
T-7559-1	1	1	3	3
T-7559-2	1	2	3	3
G-6000-1	2	1	3	3
G-6111-1	3	1	3	3
G-6111-2	3	2	3	3
G-6112-1	4	1	3	3
G-6221-1	5	1	3	3
G-7221-1	6	1	3	3
G-7222-1	7	1	3	3
G-7223-1	8	1	3	3
G-7229-1	9	1	3	3
G-7332A-1	10	1	0.5	0.5
G-7332B-1	11	1	0.5	0.5
G-7993-1	12	1	3	3
GF-5001-1	13	1	3	3
GF-5001-2	13	2	3	3

Figure 4. Data on faculty; stipend, quality rating, adjunct or full-time faculty (binary), number of courses in load for full-time faculty, full-time faculty willingness to teach course as an overload (binary)

k	s_k	q_k	adj	FT	m_k for FT	w_k for FT
1	$1,335.67	5	1	0		
2	$1,190.00	3.7	1	0		
3	$1,138.67	4.3	1	0		
4	$1,553.00	5	1	0		
5	$1,330.00	4.67	1	0		
6	$1,581.00	4.8	1	0		
7	$1,122.00	5	1	0		
8	$1,550.00	3.55	1	0		
9	$1,184.33	5	1	0		
10	$1,150.33	4.8	1	0		
11	$1,302.67	4	1	0		
12	$1,360.00	4.93	1	0		
13	$1,311.33	5	1	0		
14	$1,277.00	4.16	1	0		
15	$1,333.00	4.64	0	1	2	1
16	$1,333.33	4.2	0	1	0	1
17	$1,333.33	4.53	0	1	0	0
18	$1,250.00	4.13	0	1	0	0
19	$1,250.00	4.64	0	1	0	1

20	$1,333.33	4.5	0	1	0	1

Figure 5. Data on number of adjunct faculty, number of total faculty and number of sections per course

n_c	13
n_a	14
n_k	20
n_1	2
n_2	1
n_3	2
n_4	1
n_5	1
n_6	1
n_7	1
n_8	1
n_9	1
n_10	1
n_11	1
n_12	1
n_13	2

Analysis and Verification

For ease of analysis and verification, the results are presented in table 4.

Table 4. Model results

Course	Section	Faculty	Faculty Eligible to teach course i	q_k	m_k	w_k	o_k	Total h_i for faculty k	Salary of faculty k for course i
1	1	a_14	Yes	4.16	n/a	n/a	n/a	6	$3831
1	2	a_14	Yes	4.16	n/a	n/a	n/a	6	$3831
2	1	a_12	Yes	4.93	n/a	n/a	n/a	9	$4080
3	1	a_12	Yes	4.93	n/a	n/a	n/a	9	$4080
3	2	o_19	Yes	4.64	0	1	1	n/a	$3750
4	1	a_12	Yes	4.93	n/a	n/a	n/a	9	$4080
5	1	f_15	Yes	4.64	Yes, m_15 = 2	n/a	n/a	n/a	$0
6	1	a_5	Yes	4.67	n/a	n/a	n/a	6	$3990
7	1	a_7	Yes	5	n/a	n/a	n/a	9	$3366
8	1	a_7	Yes	5	n/a	n/a	n/a	9	$3366

9	1	a_7	Yes	5	n/a	n/a	n/a	9	$3366
10	1	f_15	Yes	4.64	Yes, m_15 = 2	n/a	n/a	n/a	$0
11	1	o_15	Yes	4.64	m_15 = 2 but f_15 already assigned to course 5 section 1 and course 10 section 1 so now is assigned overload to course 11 section 1	1	1	n/a	$666.5
12	1	a_5	Yes	4.67	n/a	n/a	n/a	6	$3990
13	1	a_3	Yes	4.3	n/a	n/a	n/a	6	$3416.01
13	2	a_3	Yes	4.3	n/a	n/a	n/a	6	$3416.01
								Total	**$48718.52 (matches Lingo Optimal Solution)**

There are a few things to note from the table.

1. Each course has one and only one faculty assignment.
2. Each faculty assigned to a course is eligible to teach that course.
3. Full-time faculty 15 is the only faculty member with room in his/her full-time university contract and the model takes advantage of this and assigns the faculty member to two courses at cost of $0.
4. All adjunct faculty assigned to courses are within the 9.75 WLU limit.

The partial result from Lingo software is in figure 6. Full model formulation and solution in Lingo has been omitted from report due to length.

Figure 6. Partial Lingo Result Output

```
Global optimal solution found.
Objective value:                              48718.52
Objective bound:                              48718.52
Infeasibilities:                              0.000000
Extended solver steps:                               0
Total solver iterations:                             0
Elapsed runtime seconds:                          0.24

Model Class:                                      PILP

Total variables:                  416
Nonlinear variables:                0
Integer variables:                416

Total constraints:                310
Nonlinear constraints:              0

Total nonzeros:                  2053
Nonlinear nonzeros:                 0
```

Variable	Value	Reduced Cost
AONE11	0.000000	4007.010
ATWO11	0.000000	3570.000
ATHREE11	0.000000	3416.010
AFOUR11	0.000000	4659.000
AFIVE11	0.000000	3990.000
ASIX11	0.000000	4743.000
ASEVEN11	0.000000	3366.000
AEIGHT11	0.000000	4650.000
ANINE11	0.000000	3552.990
ATEN11	0.000000	3450.990
AELEVEN11	0.000000	3908.010
ATWELVE11	0.000000	4080.000
ATHIRTEEN11	0.000000	3933.990
AFOURTEEN11	1.000000	3831.000
OFIFTEEN11	0.000000	3999.000
OSIXTEEN11	0.000000	3999.990
OSEVENTEEN11	0.000000	3999.990
OEIGHTEEN11	0.000000	3750.000
ONINETEEN11	0.000000	3750.000
OTWENTY11	0.000000	3999.990

Validation

The results of the report provide interesting insight on several areas. Due to many of the assumptions, it is unlikely the solution to the original problem can be fully applied to the course schedule. For example, this model does not take into consideration the meeting days/times of courses, which can prevent a more cost-effective instructor from being assigned to a course. However, the specific assignments in the model can be a helpful guide to the program director who may not be able to identify inefficiencies in the course planning procedure. Also, the optimal value of the model is a benchmark and measuring stick for how efficient the current scheduling of the program is. More research could be done to determine a fair level of tolerance for how far off the program director's planned budget is compared to the optimal solution given by the model.

There is also some concern about how well the assumptions hold in practicality. It is reasonable to question if the quality of instruction diminishes as faculty teach more courses, specifically full-time faculty overloads. Along the same lines, the model does not factor in how teaching assignments affect other service assignments for faculty members across the university. The model delivers an optimal value in USD and does not take into account other costs, such as retention of faculty members, searches for new faculty members, training and/or upkeep per faculty member as cost to the administration, etc. More research is needed to determine a strong measure for quality of instructor. This model uses student reviews to determine effectiveness of faculty member and ignores unbalanced data. Student reviews are only one aspect of quality of instruction. Other contributing factors to quality of instructors missing from the model are peer reviews and yearly evaluations. As a result, there is a slight margin of error for each q_k value and the values should be interpreted only as a proxy for the true q_k value of each instructor.

A convenient way to verify the results is to compare the integer programming solution with the actual teaching assignments made by the program director for the spring term. Table 5 summarizes the comparison.

Table 5. Verification of results. Comparison of integer programming solution to course schedule developed by Program Director.

Course	**Section**	**IP Solution - Assigned Faculty**	**Program Director Schedule - Assigned Faculty**	**Match?**
1	1	14	14	Yes
1	2	14	14	Yes
2	1	12	12	Yes
3	1	12	6	No
3	2	19	12	No
4	1	12	12	Yes
5	1	15	8	No
6	1	5	7	No
7	1	7	7	Yes
8	1	7	15	No
9	1	7	7	Yes
10	1	15	19	No
11	1	15	15	Yes
12	1	5	16	No

13	1	3	13	No
13	2	3	1	No

Seven of the sixteen assignments match. The current, iterative process for scheduling courses and assigning faculty has the potential to conceal improvements in the scheduling process and can lead to discrepancies. Other possible reasons for the difference of the integer programming solution from the program director's assignments are taking into consideration meeting days/times of courses and/or faculty data changing after the original data collection. For example, a full-time faculty member may be removed from a course in another program at the university and gain an m_k value for the program in this report. Changes at the university take place on a continuous basis so it is difficult to perfectly capture the data at one point in time. However, the model enables a convenient way to review discrepancies.

Sensitivity Analysis

Table 6 summarizes seven different scenarios for sensitivity analysis.

Table 6. Sensitivity analysis on seven scenarios represented contextually and mathematically with percentage change in results and comments.

Scenario	Model Adjustment	Mathematical Representation	Percent Change in Objective Value	Comments
1	Change adjunct faculty load limit from 9.0 WLU units to 12.75 WLU units	$\sum_{i=1}^{n\_c}\sum_{j=1}^{n\_i} h_i * a_{kij} \leq 12.75\ WLU\ for\ \forall\ n_a;$	-1.3%	Increasing the number of WLU units an adjunct faculty member can teach allows more cost-effective instructor(s) to teach more course(s)
2	Do not allow full-time faculty any courses as overload	Remove o_{kij} variables from model	+10.3%	By not allowing full-time faculty to teach 1 course as an overload, more expensive adjunct faculty must be used
3	Allow full-time faculty 2 overloads, assumes a full-time faculty member is willing to take on 2 overloads if he/she already willing to take on 1	$\sum_{i=1}^{n_i}\sum_{j=1}^{n_c} o_{kij} \leq 2\ for\ k = n_{a+1} - 20;$	-7.4%	By allowing full-time faculty to teach 2 courses as overload, more expensive adjunct faculty are not needed

4	Adjust average course quality rating from 4.25 to 4.50	$(1/16)(\sum_{i=1}^{n_i}\sum_{j}^{n\_c} q_k * a_{kij} + q_k * f_{kij} + q_k * o_{kij}) \geq 4.50$	+13.73%	More expensive to administer higher quality instruction
5	Allow co-teaching/team-teaching assignments (decision variables can be any real, non-negative number)	$f_{kij} \geq 0$ $o_{kij} \geq 0$ $a_{kij} \geq 0$	-1.1%	Model assigns combination of faculty to courses to minimize cost
6	Introduce a new full-time faculty member capable of teaching the design oriented courses in the program with an m_k value of 4, assumes q_k = 4.50 and w_k = 0.	Eligible to teach courses i= 3, 4, 5, 6, 7, 8, 9	-31.6%	Cost of full-time faculty is absorbed in yearly salary, reduces term-by-term faculty budget
7	Introducing a new full-time faculty member capable of teaching the business oriented courses in the program with an m_k value of 4, assumes q_k = 4.50 and w_k=0.	Eligible to teach courses i= 5, 6, 7, 8, 10, 11, 12	-30.5%	Cost of full-time faculty is absorbed in yearly salary, reduces term-by-term faculty budget

The results in table 6 provide insight on various scenarios for sensitivity analysis. Allowing adjunct faculty to be assigned more work-load units (WLU) and, therefore, the opportunity to teach an additional three credit lecture course, decreases cost to the budget. Not only does allowing full-time faculty the opportunity to teach two overloads decrease cost but also removing the opportunity to teach one overload for full-time faculty increases cost. Removing the constraint that the decision variables must be binary and allowing the decision variables to take on any nonnegative value enables the model to assign faculty to a fraction of a course for team-teaching possibilities. (Note: While relaxing this constraint allows the decision variables to be any real, nonnegative number, functional constraint #4 forces the decision variable(s) for a course to be equal to 1.) Team-teaching assignments to courses result in reduced costs. Many of these results rely on the assumption of certainty, where quality of instructor is held constant. However, there is concern about how the quality of instruction changes based on the number of teaching assignments for an instructor or the number of instructors assigned to a course.

The sensitivity analysis creates a fictional scenario where two full-time faculty of different backgrounds are considered as new, full-time faculty dedicated exclusively to the program. The faculty are divided into two categories: a design oriented faculty member and a business oriented faculty member. These two categories are very generic but illustrate the usefulness of the model to compare the hiring of a new full-time faculty member. The model compares any changes in the optimal solution if one new full-time faculty member is added over the other faculty member, which can affect the combination of assignments for the remaining adjunct faculty to courses. The same sensitivity analysis can be done on more specific categories tailored to different candidates' backgrounds such as business management, business marketing, fashion and garment design, manufacturing and supply chain, etc.

Alternative Model

An alternative method to solve the existing model is to eliminate the e_k variables for all adjunct and overload decision variables. In essence, all faculty become eligible to teach any course as an adjunct or overload assignment. To balance this, in place of the eligibility constraints, faculty that are not eligible to teach a course receive an exorbitant penalty (a big *M* stipend rate) if assigned to a course he/she is ineligible to teach. The *M* symbolically represents a huge positive number (Hillier and Lieberman 117). Thus, even though the model eliminates the e_k constraints for adjunct and overload faculty, the integer program is forced to avoid assigning ineligible faculty to courses by the exorbitant penalties on stipend rates. The results of this model are the same but this alternative method has 182 less constraints than the original model.

An alternative way to formulate the problem is to develop a "knapsack" model.

$$Max\ W = \sum_{i}^{n\_i}\sum_{j}^{n\_c}\frac{q_k}{s_k} * c_i * a_{kij} \text{for } n_k - n_A$$
$$+ \sum_{i}^{n\_i}\sum_{j}^{n\_c}\frac{q_k}{s_k} * c_i * o_{kij} \text{for } n_{a+1} - n_k$$

The "knapsack" problem chooses from a set of values to seek which value is most desirable for the model without violating a constraint (McMillan 425). The objective function above consists of a ratio for each faculty's quality rating over his/her stipend rate, where the larger the ratio, the more desirable the instructor is. This is the value the "knapsack" value seeks to maximize. The set of values under consideration is the calculated ratio for each faculty member. Retaining the binary integer properties of the original model, the highest ratios for each course receive a 1 for its corresponding decision variable and all else are 0. This model requires manual checking of all constraints until an optimal and feasible solution is found.

Conclusion

Formulating a mathematical model to assign faculty to courses for a program at a university is a powerful tool that provides a strategic approach to scheduling on a term-by-term basis. While there are assumptions in the model and limitations, the model serves as a benchmark for comparison to the current, iterative scheduling process and as a guide for program directors new to the job of scheduling courses at the university. Further analysis can be done on the parameters of the model to reveal the impact of potential policy and personnel changes. This proved very insightful showing that different scenarios can positively or negatively affect the adjunct budget, with changes in magnitude ranging from 1.1% to 31.6%.

The 2017 spring semester schedule created and implemented by the program director consisted of 25% full-time faculty and 75% adjunct faculty. This discrepancy is explained by the fact the program is currently in transition and is expected to hire a new, full-time professor for the program in the fall responsible for teaching four courses. After factoring in these four assignments to a full-time faculty member in future terms, there is still a large percentage of courses taught by adjunct faculty. At the university level, the spring 2017 semester schedule consists of about 52% full-time faculty and 48% adjunct faculty. The model can be extended to other programs and the university as a whole. Analysis at the university level provides the ability to evaluate financial impact of policies such as full-time faculty teaching load, course releases for full-time faculty to research and sabbaticals. Also, it is important to note fourteen of the courses in this report have a work-load unit value of 3.0 and the remaining two courses have a work-load unit value of 0.50. Depending on the method of a course at the university, work-load units range from 0.50 to 8.0 with many values in between at various intervals. The model is carefully constructed to take into consideration diverse work-load unit values and will still provide the optimal solution based on these parameters.

This report is a strong first step into the analysis of course scheduling at a university. The representation of the scheduling process in a mathematical model has tremendous value and many advantages. My suggestions for further study are below:

- Apply the model to a larger group of courses, across multiple programs that share courses to analyze how professors teaching in different programs affects scheduling
- More research in determining which courses should be offered on the schedule
- A more complete approach to budget analysis that takes into consideration total cost of the university, not just adjunct budget (i.e. full-time faculty annual salary, cost of hiring, etc.)
- More research to determine how to measure quality of instructor

Work Cited

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